Journal of Sound and Vibration (2000) **231**(4), 1175–1178
doi:10.1006/jsvi.1999.2648, available online at http://www.idealibrary.com on IDEAL®

TRANSVERSE VIBRATIONS OF A CIRCULAR PLATE OF POLAR ANISOTROPY WITH A CONCENTRIC CIRCULAR SUPPORT

R. H. GUTIÉRREZ AND P. A. A. LAURA

Institute of Applied Mechanics (CONICET) and Department of Engineering, Universidad Nacional del Sur, 8000-Bahía Blanca, Argentina

(*Received* 30 *September* 1999)

1. INTRODUCTION

Transverse vibrations of an isotropic circular plate with a concentric circular support have been studied in reference [1]. The present note constitutes an extension of reference [1] whereby a plate of polar anisotropy is considered.

Polynomial co-ordinate functions are employed to approximate axisymmetric modes of vibration of the structural system and the Rayleigh–Ritz method is used to generate the frequency determinant. The fundamental frequency coefficient is determined and then minimized following Rayleigh's optimization approach.

2. APPROXIMATE SOLUTION

Transverse vibrations of the polarly orthotropic plate shown in Figure 1 are governed by the functional [2]

$$J(W) = \iint_{\bar{P}} \left[D_r W''^2 + D_\theta \left(\frac{W'}{\bar{r}} \right)^2 + 2 D_r \nu_\theta \frac{W' W''}{\bar{r}} \right] \bar{r}\, \mathrm{d}\bar{r}\, \mathrm{d}\theta$$

$$- 2\pi D_r a \left[W''(a) + \nu_\theta \frac{W'(a)}{a} \right] W'(a) - \iint_{\bar{P}} W^2 \bar{r}\, \mathrm{d}\bar{r}\, \mathrm{d}\theta, \qquad (1)$$

subject to the boundary conditions

$$W(a) = 0, \qquad W'(a) = -\phi D_r \left[W''(a) + \nu_\theta \frac{W'(a)}{a} \right], \qquad W(b) = 0. \quad \text{(2a–c)}$$

The parameter ϕ denotes the flexibility coefficient at the plate edge and equation (2b) is the constitutive relation defining it. In the case of a clamped edge: $\phi \to 0$ and when the plate is simply supported: $\phi \to \infty$.

0022-460X/00/141175 + 04 $35.00/0

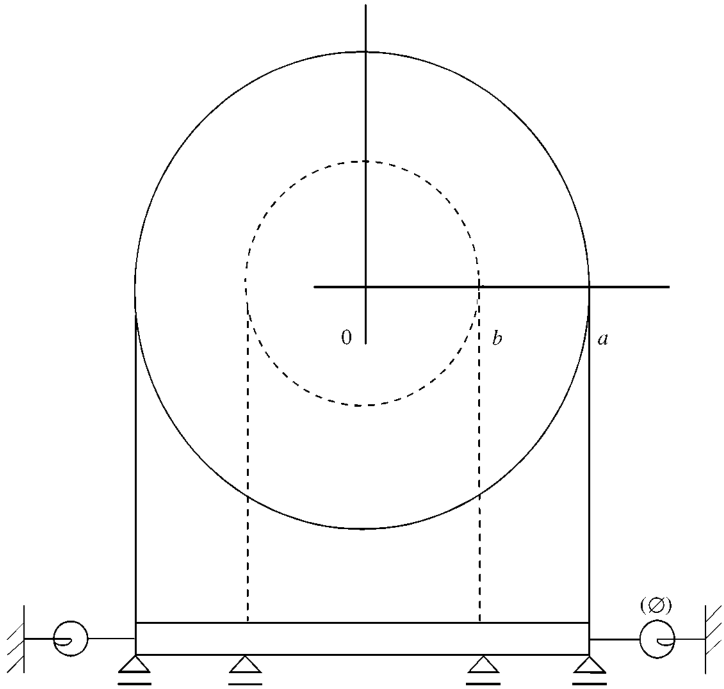

Figure 1. Vibrating system under study.

Introducing the dimensionless variable $r = \bar{r}/a$ and substituting into equations (1) and (2), one obtains

$$\frac{a^2}{2\pi D_r} J(W) = \int_0^1 \left[W''^2 + \frac{D_\theta}{D_r}\left(\frac{W'}{r}\right)^2 + 2\nu_\theta \frac{W'W''}{r} \right] r\,\mathrm{d}r$$
$$- [W''(1) + \nu_\theta W'(1)]\,W'(1) - \Omega^2 \int_0^1 W^2 r\,\mathrm{d}r, \qquad (3)$$

$$W(1) = 0, \qquad W'(1) = -\phi'[W''(1) + \nu_\theta W'(1)], \qquad W(r_b) = 0, \qquad \text{(4a–c)}$$

where $r_b = b/a$, $\phi' = \phi D_r/a$, $\Omega^2 = (\rho h a^4/D_r)/\omega^2$,

Following previous works [1, 3], one approximates the displacement amplitude $W(r)$ by means of the approximation

$$W_a = \sum_{i=1}^{N} C_j \varphi_j(r) = \sum_{i=1}^{N} C_j(a_j r^{p+j-1} + b_j r^{j+2} + c_j r^{j+1} + 1), \qquad (5)$$

where the a_j's, b_j's and c_j's are obtained by substituting each co-ordinate function in the governing boundary conditions and the exponent "p" is Rayleigh's optimization parameter.

Substituting equation (5) into equation (1), one obtains by a straightforward application of the Rayleigh–Ritz method,

$$\frac{a^2}{4\pi D_r}\frac{\partial J}{\partial C_i} = \left\{ \sum_{j=1}^{N} \int_0^1 \left[\varphi_j''\varphi_i'' + \frac{D_\theta}{D_r}\frac{\varphi_j'\varphi_i'}{r^2} + \nu_\theta \frac{\varphi_j''\varphi_i' + \varphi_j'\varphi_i''}{r} \right] r\,\mathrm{d}r \right.$$
$$- \frac{1}{2}[\varphi_j'(1)(\varphi_i''(1) + \nu_\theta\varphi_i'(1)) + (\varphi_j''(1) + \nu_\theta\varphi_j'(1))\varphi_i'(1)$$
$$\left. - \Omega^2 \int_0^1 \varphi_j\varphi_i r\,\mathrm{d}r \right\} C_j = 0. \qquad (6)$$

Equation (6) leads, finally, to a determinantal equation whose lowest root constitutes the fundamental frequency coefficient $\Omega_1 = \sqrt{(\rho h/D_r)}\,\omega_1 a^2$.

Since

$$\Omega_1 = \Omega_1(p) \qquad (7)$$

by minimizing it with respect to p one obtains an optimized value of Ω_1.

3. NUMERICAL RESULTS

The fundamental frequency coefficients of polarly orthotropic plates have been determined, for a wide range of value of b/a and D_0/D_r, by making $\nu_\theta = 0{\cdot}30$ and $N = 10$.

Tables 1 and 2 depict values of Ω_1 for simply supported and clamped orthotropic plates, respectively, as a function of D_θ/D_r and $r_b = b/a$. It is interesting to point out that in the case of isotropic plates ($D_\theta/D_r = 1$), one obtains the maximum values of Ω_1 for $b/a = 0{\cdot}4$ in both tables which correspond, roughly, to the second frequency coefficient corresponding to axisymmetric modes of the simply supported and clamped circular plate without intermediate support (29·76 and 39·77 respectively [4, 5]).

TABLE 1

Fundamental frequency coefficients of simply supported circular plates of polar orthotropy with an intermediate concentric circular support

D_θ/D_r	$r_b = 0{\cdot}1$	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·50	14·913	17·940	22·308	27·013	25·174	19·703	15·597	12·767	10·773
0·75	15·593	18·556	22·939	27·962	26·601	20·835	16·461	13·451	11·339
1·00	16·188	19·095	23·485	28·740	27·806	21·798	17·194	14·030	11·817
1·25	16·725	19·584	23·974	29·409	28·868	22·653	17·843	14·541	12·238
1·50	17·220	20·036	24·425	30·005	29·828	23·430	18·431	15·004	12·619

TABLE 2

Fundamental frequency coefficients of clamped circular plates of polar orthotropy with an intermediate concentric circular support

D_θ/D_r	$r_b = 0{\cdot}1$	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
0·50	23·548	28·274	34·531	36·205	27·709	20·807	16·238	13·141	10·945
0·75	24·306	28·952	35·330	37·922	29·301	21·967	17·119	13·837	11·518
1·00	24·966	29·539	35·995	39·324	30·653	22·953	17·865	14·426	12·002
1·25	25·561	30·069	36·578	40·521	31·850	23·827	18·524	14·946	12·429
1·50	26·109	30·557	37·103	41·571	32·936	24·620	19·121	15·417	12·815

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program and by Secretaria General de Ciencia y Tecnología of Universidad Nacional del Sur.

REFERENCES

1. P. A. A. LAURA, R. H. GUTIÉRREZ, V. H. CORTÍNEZ and J. C. UTJES 1987 *Journal of Sound and Vibration* **113**, 81–86. Transverse vibrations of circular plates and membranes with intermediate supports.
2. S. G. LEKHNITSKII 1968 *Anisotropic Plates*. New York, NY: Gordon and Breach, Science Publishers, Ltd.
3. R. H. GUTIÉRREZ, P. A. A. LAURA and R. E. ROSSI 1999 *Journal of Sound and Vibration*. Vibrations of circular annular plate of cylindrical anisotropy and non-uniform thickness (to be published).
4. A. W. LEISSA 1969 *Vibration of Plates*. NASA SP 160.
5. P. A. A. LAURA 1999 *Journal of Sound and Vibration* **222**, 853–856. Comments on "Optimal support positions for a structure to maximize its fundamental natural frequency".